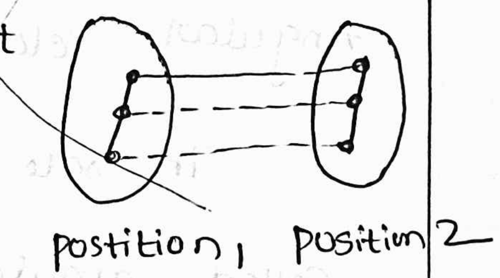
Types of motion

A particle can perform only translation but a rigid body can perform any motion.

1. Translation motion.
2. Rotational motion
3. Plane motion.

Translation motion

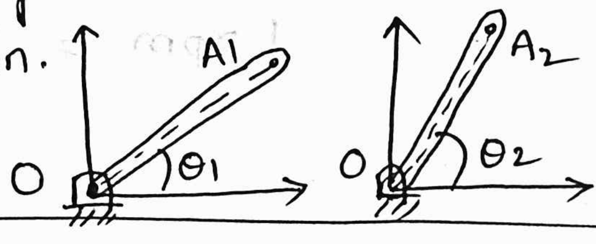
A body is said to perform translation motion if an imaginary straight line drawn on the body remains parallel to original position during its motion at any instant



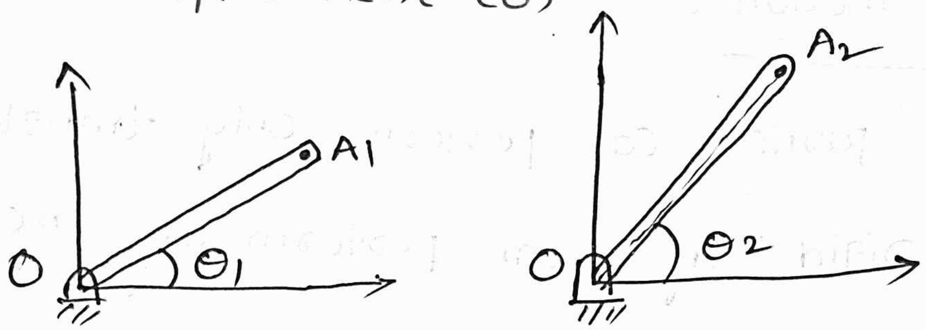
1. Rectilinear Translation
2. Curvilinear Translation

Fixed axis rotational motion

In fixed axis rotation the centre of rotation and axis is stationary therefore such motion is called fixed axis rotation.



Angular Displacement (θ)



If θ_1 is the angular position of the body in position ① and it changes to θ_2 at position 2 then angular displacement of the body θ is given as

Angular displacement (θ) = final angular position − Intial Angular position

$$\theta = \theta_2 - \theta_1$$

Angular velocity (ω)

The rate of change of angular position is called angular velocity of rotation.

$$\omega = \frac{d\theta}{dt}$$

- Angular velocity is measured in rad / sec

$$1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

Angular Acceleration (α)

The rate of change of angular velocity is called angular acceleration.

$$\alpha = \frac{d\omega}{dt} \quad rad/s^2$$

Displacement, velocity and acceleration relation for fixed axis rotation.

Translation motion	Rotational motion
$v = u + at$	$\omega = \omega_0 + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 - u^2 = 2as$	$\omega^2 - \omega_0^2 = 2\alpha\theta$

13.3 Relationship Between Rope, Pulley and Block

Figure 13.3-i shows a pulley of radius r mounted on pin. Block A is connected by rope which is wound around the pulley. The rotational motion of pulley will relate to translation motion.

Consider at given instant the pulley has an angular position (θ), angular velocity (ω) and angular acceleration (α).

The block A will have corresponding position (x_A), velocity (v_A) and acceleration (a_A) at this instant.

Let P be the common point between pulley and rope. Here pulley is performing rotational motion and block is performing translation motion. Since P is the common point of contact between both pulley and rope, we have

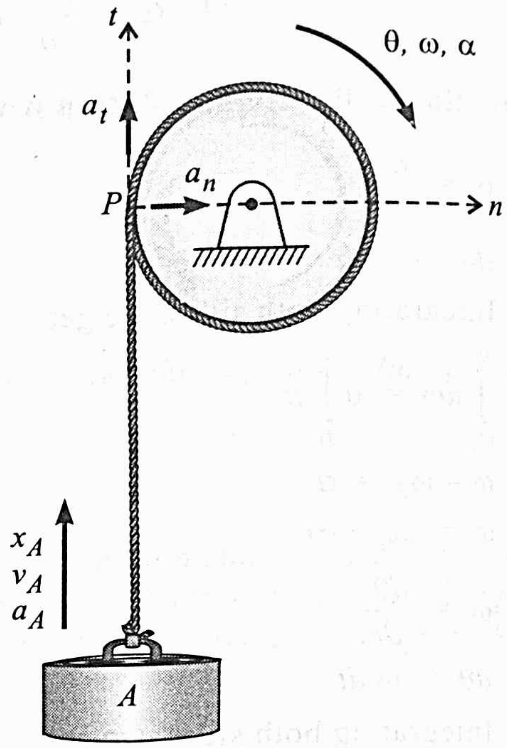


Fig. 13.3-i

$$x_P = x_A = r\theta$$

$$v_P = v_A = r\omega$$

$$a_P = a_A = r\alpha$$

{a_A is the component of acceleration along tangential direction a_t}

$$a_n = \frac{v^2}{r}$$

{a_n is the component of acceleration along normal direction a_n}

13.4 Relationship Between Two Contact Pulleys Rotating without Slipping

Consider pulley ① of radius r_1 and pulley ② of radius r_2 mounted on pins rotating as shown in Fig. 13.4-i without slipping.

At a given instant let pulley ① have angular position θ_1, angular velocity ω_1 and angular acceleration α_1. Let pulley ② have angular position θ_2, angular velocity ω_2 and angular acceleration α_2.

Let P be the common point of contact. If pulley ① rotates clockwise, pulley ② will rotate anticlockwise.

Assuming point P on pulley ①, we have the following relationship

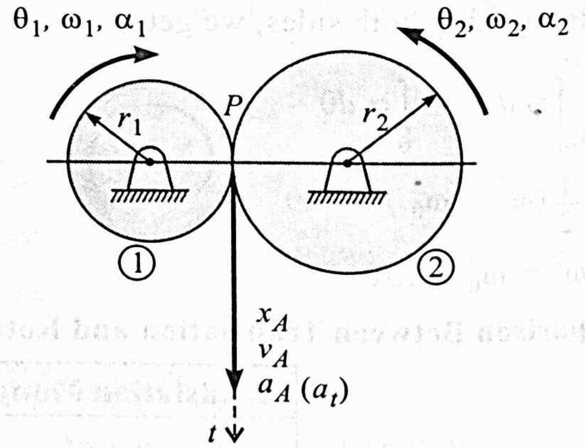


Fig. 13.4-i

$$\left.\begin{aligned} x_1 &= r_1\theta_1 = x_P \\ v_1 &= r_1\omega_1 = v_P \\ a_1 &= r_1\alpha_1 = a_P\ (a_t) \end{aligned}\right\} \quad \ldots (13.1)$$

13.5 General Plane Motion

General plane motion is the combination of translation motion and rotational motion happening simultaneously.

Example

Consider a rod AB having one end A on vertical wall and other end B on floor is sliding. Therefore, velocity of point A (v_A) will be vertically down and that of point B (v_B) will be horizontally towards right.

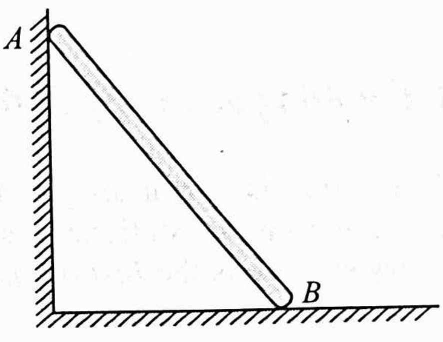


Fig. 13.5-i(a)

Drawing perpendicular to direction of v_A and v_B we get centre I. Hence, I is the ***Instantaneous Centre of Rotation (ICR)***.

Why *I* is called ICR ?

I is called Instantaneous Centre of Rotation because the velocity of point A and B at an instance is giving I. It means at some other instant rod AB will have position $A'B'$ and velocities v_A' and v_B'. Therefore, its ICR will be I'.

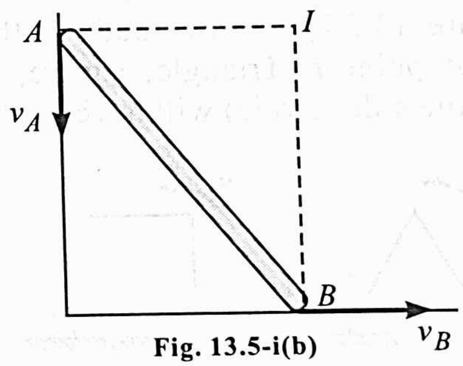


Fig. 13.5-i(b)

How to identify General Plane Motion ?

In this example, we have rod AB slipping against vertical wall and horizontal floor. AB is the one position after some next instant $A'B'$ is another position. Here we observe that the rod had shifted its position, means there is translation motion involved, also the angle of rod had changed means there is rotational motion. This is happening simultaneously. Therefore, rod is performing general plane motion.

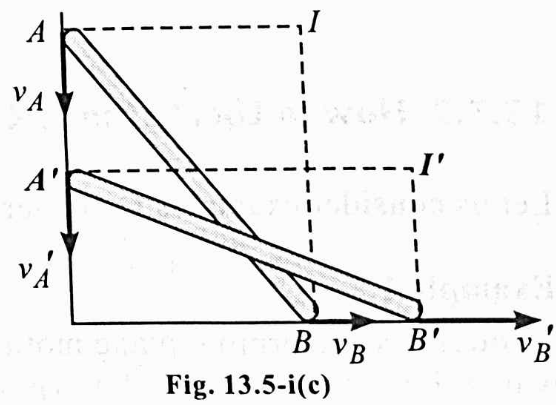


Fig. 13.5-i(c)

Note : When a body is performing general plane motion, it is assumed to perform fixed axis rotation about some centre say I. This centre of roatation changes its position instant to instant. Therefore, it is called as *Instantaneous Centre of Rotation (ICR)*.

13.6 Solved Problems

Problem 1

A fly wheel starts from rest and after half a minute rotates at 2000 rpm. Calculate the **(i)** angular acceleration and **(ii)** number of revolution made by the wheel within this period.

Solution

$$\omega_0 = 0 \qquad \omega = \frac{2\pi \times 2000}{60}$$

$$t = 30 \text{ sec} \qquad \omega = 209.44$$

(i) $\omega = \omega_0 + \alpha t$

$$209.44 = 0 + \alpha \times 30$$

$$\alpha = 6.98 \text{ rad/s}^2$$

(ii) $\theta = \omega_0 t + \frac{1}{2}\alpha \times t^2$

$$\theta = 0 + \frac{1}{2} \times 6.98 \times 30^2$$

$$\theta = 3141 \text{ rad}$$

$$\text{Number of revolution } n = \frac{\theta}{2\pi} = \frac{3141}{2\pi}$$

$n = 500$ ***Ans.***

Problem 2

A rotor of turbine has an initial angular velocity of 1800 rpm. Accelerating uniformly, it doubled its velocity in 12 sec. Find the revolutions performed by it in this interval.

Solution

$$\omega_0 = \frac{2\pi N}{60} = \frac{2\pi \times 1800}{60}$$

$$\omega_0 = 60\pi \text{ rad/sec} \text{ and } \omega = 2\omega_0 = 120\pi \text{ r/s}$$

$$t = 12 \text{ sec}$$

$$\omega = \omega_0 + \alpha t$$

$$120\pi = 60\pi + \alpha(12)$$

$$\alpha = 5\pi \text{ r/s}^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\theta = 60\pi \times 12 + \frac{1}{2} \times 5\pi \times 12^2$$

$$\theta = 1080\pi \text{ rad}$$

$$\text{Number of revolutions } n = \frac{\theta}{2\pi} = \frac{1080\pi}{2\pi}$$

$n = 540$ ***Ans.***

Problem 10

A motor gives disk A an angular acceleration of $\alpha_A = (0.6t^2 + 0.75)$ rad/sec^2, where t is in seconds. If the initial angular velocity of the disk is $\omega_0 = 6$ rad/sec, determine the magnitude of the velocity and acceleration of block B when $t = 2$ sec.

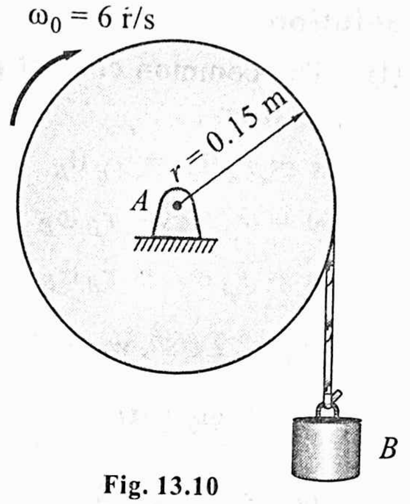


Fig. 13.10

Solution

At $t = 2$ sec

$$\alpha = 0.6t^2 + 0.75 = 0.6 \times 2^2 + 0.75$$

$$\alpha = 3.15 \text{ rad/sec}^2$$

$$a_B = r\alpha = 0.15 \times 3.15$$

$$a_B = 0.4725 \text{ m/s}^2$$

$$\alpha = 0.6t^2 + 0.75$$

$$\frac{d\omega}{dt} = 0.6t^2 + 0.75$$

$$d\omega = (0.6t^2 + 0.75)\, dt$$

Integrating both sides, we have

$$\int d\omega = \int (0.6t^2 + 0.75)\, dt$$

$$\omega = \frac{0.6t^3}{3} + 0.75t + c_1$$

At $t = 0$, $\omega = 6$ r/s $\therefore c_1 = 6$

$$\omega = 0.2\, t^3 + 0.75 \times t + 6$$

At $t = 2$ sec

$$\omega = 0.2 \times 2^3 + 0.75 \times 2 + 6 = 9.1 \text{ rad/sec} \quad \textit{Ans.}$$

$$v = r\omega$$

$$v_B = 0.15 \times 9.1 = 1.365 \text{ m/sec} \quad \textit{Ans.}$$

Problem 11

Pulley A starts from rest and rotates with a constant angular acceleration of 2 r/s^2 anticlockwise. Pulley A causes double pulley B to rotate without slipping. Block C hangs by a rope wound on B, refer to Fig. 13.11(a). Determine at $t = 3$ sec.

(i) Acceleration, velocity and position of block C.

(ii) Acceleration of point P on pulley B.

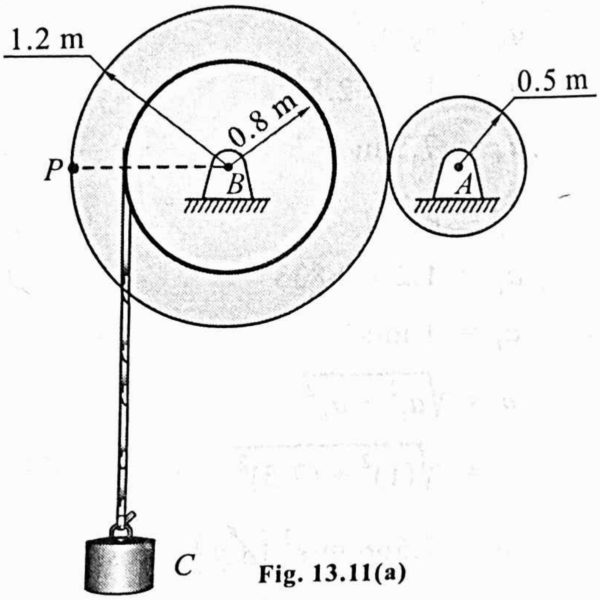


Fig. 13.11(a)

Solution

(i) The common contact point between double pulley B and pulley A will have the following relation :

$$s = r_A \theta_A = r_B \theta_B$$
$$v = r_A \omega_A = r_B \omega_B$$
$$a = r_A \alpha_A = r_B \alpha_B$$

$$\alpha_A = 2 \text{ r/s}^2, \; w_0 = 0, \quad t = 3 \text{ sec}$$

$$\omega_A = \omega_0 + \alpha t \qquad \theta_A = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega_A = 0 + 2 \times 3 \qquad \theta_A = 0 + \frac{1}{2} \times 2 \times 3^2$$

$$\omega_A = 6 \text{ r/s} \qquad \theta_A = 9 \text{ rad}$$

$$\therefore \omega_B = \frac{r_A \omega_A}{r_B} = \frac{0.5 \times 6}{1.2} \qquad \therefore \theta_B = \frac{r_A \theta_A}{r_B} = \frac{0.5 \times 9}{1.2}$$

$$\therefore \omega_B = 2.5 \text{ r/s} \qquad \therefore \theta_B = 3.75 \text{ rad}$$

$$\therefore \alpha_B = \frac{r_A \alpha_A}{r_B} = \frac{0.5 \times 2}{1.2}$$

$$\therefore \alpha_B = 0.833 \text{ r/s}^2$$

The common contact point between double pulley B (inner radius 0.8 m) and rope connected to block C will have the following relations :

$$s_C = r_B \theta_B \;; \qquad v_C = r_B \omega_B \;; \qquad a_C = r_B \alpha_B$$

$$s_C = 0.8 \times 3.75 \qquad v_C = 0.8 \times 2.5 \qquad a_C = 0.8 \times 0.833$$

$$s_C = 3 \text{ m} \qquad v_C = 2 \text{ m/s} \qquad a_C = 0.67 \text{ m/s}^2$$ ***Ans.***

(ii) Acceleration of point P

$$a_n = r_B \omega_B^2$$
$$a_n = 1.2 \times 2.5^2$$
$$a_n = 7.5 \text{ m/s}^2$$

$$a_t = r_B \alpha_B$$
$$a_t = 1.2 \times 0.833$$
$$a_t = 1 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$
$$= \sqrt{(1)^2 + (7.5)^2}$$

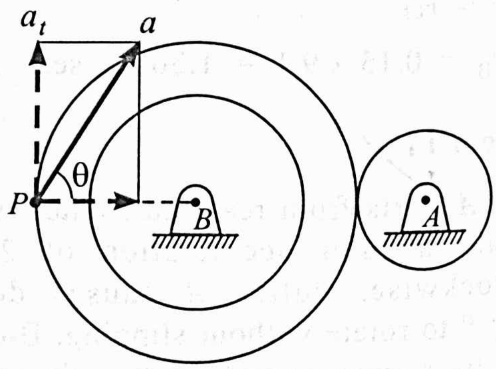


Fig. 13.11(b)

$$a = 7.566 \text{ m/s}^2 \; (\measuredangle \theta)$$ ***Ans.***

$$\tan\theta = \frac{a_t}{a_n} \qquad \therefore \theta = 7.595^\circ$$ ***Ans.***